A new theorem for optimizing the advertising budget

Wright, Malcolm
Ehrenberg-Bass Institute, University of South Australia

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Malcolm Wright

Ehrenberg-Bass Institute for Marketing Science

malcolm.wright@marketingscience.info

GPO Box 2471, Adelaide

SA 5001, Australia

Ph +61-8-8302 7640

Fax +61-8-8302 0442

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This is a working paper. Feedback is solicited.

A New Theorem for Optimizing the Advertising Budget

Abstract

This paper reports a new theorem and proof for optimizing the advertising budget. The theorem is that the optimal rate of advertising is equal to gross profit multiplied by advertising elasticity. This does not involve a ratio of elasticities, and so is an advance on the Dorfman-Steiner theorem that has dominated this topic for the last 50 years. The elegant nature of the proof makes it especially suitable for managerial economics textbooks. The simple nature of the theorem means that it is easily adopted, by both large and small businesses, in place of heuristics such as industry advertising to sales ratios. From meta-analysis, the mean advertising elasticity is .11. Therefore, in the absence of any other information, companies should spend 11% of gross profit on advertising.

Introduction

This paper reports a new theorem and proof for optimizing the advertising budget. The theorem is extremely simple, and represents an advance over previous advertising optimization methods in both marketing and economics. The Dorfman-Steiner theorem (1954) of advertising optimisation uses a linear functional form and relies on the advertising to sales ratio being equivalent to the ratio of advertising elasticity to price elasticity. Brook (2005) recently demonstrated that, when using an independent linear demand specification for advertising and price (or quantity), the Dorfman-Steiner equation reduced to an identity, implying that it is of little practical use. The new theorem does not assume linearity, offers a result that is more easily interpretable by managers and relaxes the Dorfman-Steiner requirement that advertising and price be dependent. Thus, it overcomes Brook's (2005) objections to the Dorfman-Steiner theorem.

Assumptions

(1) Profit = G - A

(1a) G = (P – C) Q

Where
P is unit price.
C is unit variable cost.
Q is quantity demanded, defined below in (2).
A is advertising cost.

(2) $Q = k A^{e}$

Where k = constant (1 for the purposes of this analysis).

e = the exponent for the marketing mix element, in this case advertising (known through a simple algebraic proof to be the elasticity of Q to changes in that element).

Note the assumption of a multiplicative function in equation (2): while empirical studies of supermarket checkout scanner data do not generally identify one functional form as being superior to others in marketing mix modelling (e.g. Bolton 1989), multiplicative functions are easily interpretable by managers as they have a constant elasticity. They are also widely used: see Danaher, Bonfrer and Dhar (2008) for a recent example, in which the multiplicative functional form is justified to avoid aggregation bias. The use of this functional form is the major assumption in this work; however the elegance of the proof and avoidance of problems arising from linearity lend it some support.

Theorem: Profit is maximised when advertising expenditure equals gross profit times advertising elasticity

(Note: This theorem was discovered through simulation. The full simulation method is reported in early versions of this working paper.)

Proof

Profit	=	$G - A$	from (1)
	=	$(P - C)\, Q - A$	by substitution of (1a)
	=	$(P - C)\, k\, A^{e} - A$	by substitution of (2)

$$\frac{d\,\text{Profit}}{d\,A} = e\,(P - C)\, k\, A^{e-1} - 1$$

To maximise profit we set this differential equal to zero, add one to both sides, and then multiply both sides by A. This yields.

$$A = e\,(P - C)\, k\, A^{e-1} * A$$

Giving	A	$=$	$e (P – C) k A^{e}$		
That is also	A	$=$	$e (P – C) Q$	from (2)	
Or	A	$=$	$e G$		
Or	A / G	$=$	e	to optimise profit.	**Q.E.D.**

Although this is a proof for advertising, it can equally be applied to any marketing mix element that has a positive elasticity, as such an item (for example distribution expenditure) could simply be substituted for A in (2). However, the theorem does not address interactions between marketing mix elements.

Discussion

Rather than use the advertising to sales ratio, and assume a linear dependence between price and advertising, this theorem implies that advertising budgets should be set so the advertising/gross profit ratio is equal to the advertising elasticity. This is a new budgeting rule for advertising expenditure, simple enough to be applied by any business that can calculate a gross profit. From meta-analysis, we know that the average advertising elasticity is .11 (Sethuraman and Tellis 1991). Thus, in the absence of any other information firms should spend 11% of gross profit on advertising. If gross profit is negative, firms should instead close their doors, as they are not covering variable costs, let alone overheads.

In practice, this is of course overly simplistic. There may be both short and long term advertising effects, or advertising effectiveness may vary under different circumstances or with different advertising executions. Conceptually, however, many of these issues can be dealt with by adjusting the advertising elasticities. Here are four examples.

First, if we accept the BehaviorScan results that the average advertising elasticities are .26 for new products (for three years after launch) and .05 otherwise (Lodish et al. 1995), we can optimize advertising under each circumstance. In this case advertising expenditure should be 26% of anticipated gross profit for the first three years post-launch, dropping back to a maintenance budget of 5% of gross profit thereafter. (While the sample of BehaviorScan experiments is large (n = 389) they are conducted using a single method, so their elasticities are less generalizable than the multi-method meta-analysis of Sethuraman and Tellis 1991).

Second, we can apply Danaher et al.’s (2008) work on clutter. They found pure advertising elasticities of .16, compared to .076 under conditions of high clutter (note that the average of these values is .12, close to that of the meta-analysis). Danaher et al. recommended scheduling advertising to avoid competitive interference. It was not clear what would happen everybody followed this advice, as they did not calculate a game-theoretic equilibrium. We can extend their recommendations to find a budget equilibrium (although not a scheduling equilibrium) without resorting to game theory. The equilibrium advertising budget under conditions of high clutter, in the markets Danaher et al. studied, is simply 7.6% of gross profit.

Third, the theorem can guide the advertising budget during a recession. Businesses are prone to slash advertising at such times; the theorem developed in this paper suggests that advertising should be cut to the extent that gross profits are expected to be cut, but no further.

Fourth, what if a creative execution is particularly effective? More effective advertising implies a higher elasticity for advertising expenditure. Therefore, effective creative executions should be backed by a higher advertising budget – as long as this does not promote earlier wearout of effectiveness.

While the new theorem is extremely simple, many enhancements are possible to relax assumptions and develop more complex models of market behavior. These could include introducing game-theoretic competition, splitting out long-term and short-term effects, modelling interactions and incorporating tactical elements such as price promotions, media buying decisions and message strategy. Actual advertising policies could also be investigated, to see whether theoretically sub-optimal decisions lead to lower firm profits. The theorem could be developed to include other advertising measures, such as an adstock decay variable, or a competitive share of voice. New dependent variables could be developed that are closer to the true objective of most firms: increasing shareholder wealth or market to book ratios. The theorem provides an elegant and productive line of enquiry for investigating such issues; they can be framed as relaxations of the basic assumptions made in (1) and (2).

Meanwhile, a small business with limited expertise or resources currently has no sound rule available to optimize advertising expenditure. This new theorem provides such a rule. It is so simple that the theorem itself can be used as a heuristic by anybody capable of basic arithmetic. In the absence of any other information, small businesses should spend 11% of their gross profits on advertising.

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